Bibliographie

N. Bourbaki, Éléments de mathématique. Fasc. XVII: **Théorie des ensembles.** Chapitre I: Description de la mathématique formelle. Chapitre II: Théorie des ensembles (Actualités scientifiques et industrielles, No.1212), third revised and corrected edition, iv+141 pages (1 foldout), Paris, Hermann, 1966.

The original edition appeared in 1954 and was the first systematic treatment of the elements of set theory from an axiomatic angle. Though the book does not presuppose any preliminary knowledge, it may be advisable that one should not start reading without a minimal background in intuitive set theory and, perhaps, in logics. In evaluating the presentation of the material one should take the governing principles of the Bourbaki series into account. According to these, the treatment of the subject is very abstract, and the aim of obtaining quickly the largest possible number of results dominates over didactical considerations. Nevertheless, the examples interspersed in the text largely extenuate the problems arising from this, particularly if the reader has some background knowledge of the subject. The style is clear but one cannot help mentioning the slight overuse of symbols. Incidentally, the most outrageous example of this in the first edition, the pairing symbol $\supset$, has been eliminated. This occurred only four times in the whole volume, and did not play an important role.

The first chapter presents a variant of first order functional calculus with equality. Hilbert's ε (here denoted by τ) is introduced along with another sign $\square$; this latter symbol may be used to eliminate variables from formulas. Formal systems are dealt with; here the interesting feature is that the notion of theorem is described as contingent upon the actual knowledge that a formal proof exists. The Appendix, inserted between the two chapters of the book, characterizes the terms and relations of a formal theory with the aid of free semigroups. Finally, the second chapter describes a variant of Zermelo—Fraenkel's set theory supplemented with the abovementioned symbol τ. Here stress is laid on the extensive treatment of elementary notions like those of relation (here called graph), function, union, intersection, complement, Cartesian product, and equivalence relation.

The excercises, which in the first edition were placed at the end of larger sections, have been removed to the end of the two chapters of the book.

A. Máté (Szeged)

N. Bourbaki, Éléments de mathématique. Fascicule XXI, livre V: **Intégration.** Chapitre V: **Intégration des mesures,** 2ième édition, revue et corrigée, 154 pages, Paris, Hermann, 1967.

Dans ce chapitre du livre sur l'intégration on considère la relation

$$\int_X f(s)\,d\mu(s) = \int_T d\nu(\lambda) \int_X f(s)\,d\mu_\lambda(s),$$

où T et X sont des espaces localement compacts, μ et $\mu_\lambda (\lambda \in T)$ des mesures sur X, ν une mesure sur T et f une fonction sur X. On démontre dans une grande généralité le théorème de Lebesgue—Nikodým, le théorème de Lebesgue sur la décomposition des mesures en leurs partie absolument continue et partie singulière, le théorème sur la dualité des espaces L^p, le théorème sur le changement de variable dans l'intégrale de Lebesgue, et le théorème de Lebesgue—Fubini sur les intégrations successives. Comme c'est général dans la série Bourbaki, on ajoute au texte un grand nombre d'exercises. Il y a une note historique relative aux chapitres II—V. A la fin on trouve une table de concordance de la première et de la seconde édition de ce livre. On suppose une connaissance des chapitres précédents.

J. Szűcs (Szeged)

Mathematical Aspects of Computer Science, Proceedings of Symposia in Applied Mathematics, Vol. XIX) edited by **J. T. Schwartz**, V+224 pages, American Mathematical Society, Providence, Rhode Island, 1967.

The book contains abstracts of lectures held at a symposium of the American Mathematical Society on mathematical aspects of computer science in New York City in 1966.

The volume containing eleven papers comprises a wide circle of problems of the sort announced in the title. J. A. Robinson gives a well-selected review on automatic theorem-proving. R. W. Floyd outlines a variant of formal definitions of the meanings of programs in appropriately defined programming languages, so that a rigorous standard is established for proofs about computer programs, including proofs of correctness, equivalence and termination. The paper of J. McCarthy and J. Painter contains a proof of correctness of a simple algorithm for compiling arithmetic expressions into machine language. The study of J. Hartmanis deals with the close relationship between context-free languages and Turing machine computations. S. Kuno's paper is a survey containing a large material on computer analysis of natural languages. In his paper M. O. Rabin gives a survey on the most important aspects of algebraic automata theory. The central theme of the study of M. Minsky and S. Papert is an appropriate classification of certain geometrical properties according to the type of computation necessary to determine wether a given figure has them. Some further papers (P. Swinnerton Dyer, M. E. Mahowald and M. D. MacLaren, C. E. Leith, J. F. Traub) are devoted to the special applications of computer technique in different fields of sciences.

I. Peák (Szeged)

L. S. Pontriagin, A course in ordinary differential equations (International Monographs on Advanced Mathematics and Physics), X+333 pages, Delhi, Hindustan Publishing Corporation (India), 1967.

The monograph under review was first published in Russian in 1961 and is based upon lectures given by the author at Moscow State University. His point of view and his organization of the material, though preserving the essential classical features, differ often from what is usual, above all in paying special attention to the theory of oscillations and of automatic control, which seem to have the most important applications in engineering.

Thus the most exciting part of the work is the chapter discussing stability questions, and giving a full discussion of the behavior of the Watt regulator and of the applications to vacuum-tube circuits by Andronov.

Much of this material is not easily accessible in the original, hence its inclusion in the book is welcome.

Written in a lucid style and omitting some refinements which have only theoretical importance (e.g. the Lipschitz condition), it gives also the main ideas of many complicated proofs. So the proofs are presented in a very clear and natural way. It is a pleasure not to meet here the vague phrases "it is easy to see" or "by well-known methods we have", which in many other books appear and sometimes require from the reader a lot of work of clarification.

In summary the book can particularly be recommended to those who wish to be acquainted with the main classical and modern results, methods, and applications of ordinary differential equations.

There are six chapters: 1. Introduction. 2. Linear equations with constant coefficients. 3. Linear equations with variable coefficients. 4. Existence theorems. 5. Stability. 6. Certain results from linear algebra.

L. Hatvani—L. Pintér (Szeged)

V. I. Arnold—A. Avez, Ergodic problems of classical mechanics (Mathematical Physics Monograph Series), IX+286 pages, New York—Amsterdam, W. A. Benjamin, Inc., 1968.

This valuable book contains four Chapters and 34 Appendices, the latter being devoted to examples and detailed proofs of the theorems occurring in the text.

Chapter 1 (Dynamical Systems) introduces the notions of classical and abstract dynamical systems. The classical dynamical system is a smooth manifold M with a measure μ on M defined by a continuous density and a one-parameter group of measure preserving diffeomorphisms Φ_t of M. An abstract dynamical system is a measure space (M, μ) equipped with a one-parameter group $\{\Phi_t\}$ of authomorphisms $(\operatorname{mod} \mu)$ of (M, μ). In both cases Φ_t depends measurably on t, where t varies over the reals or the integers. In this chapter and in the Appendices we find several examples of dynamical systems such as the geodesic flow of the torus, the pendulum motion, Bernoulli schemes, and the so-called baker's transformation.

Chapter 2 (Ergodic properties) begins with the individual ergodic theorem of G. D. Birkhoff. It continues with ergodicity, mixing properties, spectral invariants, the discrete spectrum theorem, Lebesgue spectrum, K-systems, and entropy. The authors prove that Bernoulli systems are K-systems and sketch a proof of the assertion that K-systems have denumerably multiple Lebesgue spectrum. This chapter contains the famous theorem of Kolmogorov: The entropy of an automorphism is equal to the entropy of a generator with respect to this automorphism. The authors also prove that the entropy of K-systems is positive and establish the theorem of Kouchirenko: Classical dynamical systems have finite entropy. In the Appendices to this chapter proofs of theorems occurring in the text and other questions are studied, such as the fine analysis of the convergence of the averages, the ergodic properties of the translations of tori, conditions for classical dynamical systems to be ergodic, the asymptotic distribution of the first digits of 2^n, the mean motion of the perihelion, the notion of skew-products of dynamical systems and that of the conditional entropy of arbitrary measurable partitions, the discrete spectrum of classical dynamical systems, and an example for a mixing endomorphism.

Chapter 3 (Unstable systems) studies highly unstable systems, i.e. systems for which two orbits with close initial data are exponentially divergent. Here they are called C-systems, with a preference for Anosov's terminology to the name U-systems, widespread in the English literature. The unstability property implies the asymptotic independence of past and future, and hence C-automorphisms are ergodic, mixing, have Lebesgue spectrum, positive entropy, and, in general, are K-automorphisms. In this chapter and in its Appendices we find the definition of a C-diffeomorphism and of a C-flow with illustrative examples, the theorem of Lobatchewsky and Hadamard on the geodesic flow of compact, connected Riemannian manifolds of negative curvature, Sinai's theorem on the two foliation of a C-system, and the question of structural stability and ergodic properties of C-systems. This chapter also contains Anosov's theorem on the structural stability of C-systems. The proofs are also elaborated in the text or in the Appendices. The authors remark that with the aid of the generalization of the theory of C-systems Sinai proved the Boltzmann—Gibbs ergodic conjecture. The proof is omitted because it would require "hundreds of pages".

Chapter 4 (Stable systems) studies systems "the orbits of which, with remarkable stability, fail to fill up the 'energy level' $H=Ct$ ergodically and remain (to the end) in their particular corner of phase-space". The systems that are close to an integrable one and the systems to which the theory of perturbations of Celestial Mechanics applies are such systems. The authors report on the elements of the theory of these questions and on the present state of these problems, thereby giving an introduction to the famous works of Siegel and Kolmogorov, which gave the essential impetus to remove the theory from its previous resting point.

The book is a translation from the French edition (1967) and is based on lectures delivered during the spring and fall of 1965 by V. I. Arnold. This translation contains also new results which have appeared since the French edition. The work supposes some familiarity with differential geometry, measure theory and classical mechanics, and is written for physicists, astronomers, mathematicians, graduate students and research workers in these fields. It is well readable and contains an index and a medium-size bibliography.

J. Szűcs (Szeged)

Jun-iti Nagata, Modern general topology (Bibliotheca Mathematica, Vol. 7), VIII+353 pages, Amsterdam, North-Holland—Groningen, Wolters-Noordhoff, 1968.

The book, as is claimed in the introduction, is intended to serve both as a textbook and as a reference book. The way these two objectives are simultaneously achieved is the following: Though a large amount of material is amassed in the book, it is so arranged that different groups of readers may have their choice; various instructions given in the introduction are helping the reader in what to choose to read first and what to skip or put off for a later occasion. The double aim of the book also influenced the way in which the material is presented. Namely, priority is given to concrete and vivid methods against the perhaps more effective, but more abstract methods, which are less digestible at least for the beginner.

The book is divided into seven chapters, each of which ends with intstructive (and not too difficult) excercises, and contains an extensive bibliography. The chapter headings are the following: I. Introduction, II. Basic concepts in topological spaces, III. Various topological spaces, IV. Compact spaces and related topics, V. Paracompact spaces and related topics, VI. Metrizable spaces and related topics, VII. Topics related to mappings.

Chapter I develops the basic tools of set theory necessary for later discussions. Cardinal and ordinal numbers are introduced, and Zermelo's "Wohlordnungssatz" and Zorn's lemma are established. (The inclusion of these in the material clearly demonstrates that no preliminary factual knowledge is necessary for reading this book.) Finally, a description of the topology of the Euclidean plane is given.

Chapter II describes the underlying concepts of topology such as neighborhood, closure, convergence of filters and nets. Various kinds of space obtained form other spaces are considered: e.g. product space, quotient space, and inverse limit space. The chapter concludes with a study of connectedness.

Chapter III deals with spaces satisfying various degrees of separation (T_1, T_2, regular, completely regular, normal, and fully normal spaces), introduces compact, paracompact, and metric spaces, and various axioms of countability are described. In this context Urysohn's imbedding theorem and Baire's category theorem are proved.

The first section of Chapter IV contains Tychonoff's theorem on the compactness of products of compact spaces and some of its applications, e.g. the derivation of the Stone—Čech compactification theorem. In a new section, compactification is discussed at length, Shanin's theorem is proved. The lattice of bounded continuous functions on compact and completely regular spaces is studied, and various extensions of the concept of compactness are considered (sequential compactness, countable compactness, pseudo-compactness, Q-spaces).

Chapter V is a rather detailed account of paracompact spaces. Stone's fundamental theorem opens the row, further miscellaneous properties are investigated, countably paracompact spaces are discussed, and various other modifications of the concept of paracompactness are presented (strong paracompactness and collectionwise normality). In the last section here paracompact T_2 spaces are characterized as those spaces the product of which with any compact T_2 space is normal (Tamano).

Chapter VI starts with several comprehensive metrization theorems, among them Alexandroff—Urysohn's and Nagata—Smirnoff's, and Čech's theorem on metrizability with a complete metrics; then imbedding of metrizable spaces into given concrete metric spaces, metrizability questions involving the image and union of metric spaces are studied. Various extensions of metric spaces (uniform spaces, proximity spaces, P-spaces) are discussed at considerable length.

Chapter VII is a medley of topics related to mappings. It starts with the classical ways of introducing topologies on the set of mappings from one space into another, like weak topology and strong topology (these are familiar from the theory of bounded linear Banach-space operators), compact open topology, topology of uniform convergence. The discussion continues with rather new subjects, like how a continuous mapping transfers certain kinds of spaces (metric or paracompact) and how to characterize a space as the inverse limit space of simpler spaces; and finally a rather abstract account is given of the theory of extension of continuous mappings.

The style of the book is clear; the understanding of the text is facilitated by the many interspersed examples. The publisher should also be praised for contributing to the intrinsic value of this book by supplying it with a neat form.

A. Máté (Szeged)

S. Sternberg, Celestial Mechanics, Part I (Mathematics Lecture Notes Series), XXI+158 pages, New York—Amsterdam, W. A. Benjamin, Inc., 1969.

This book is the first part of a very sketchy survey of the historical developments of astronomy in an up-to-date and self-contained presentation with valuable mathematical material. It does not contain the results of recent works of Kolmogoroff, Arnold and Moser. These results will be contained in Part II.

Chapter 1 starts with the notion of almost periodic function as an idealization of the motion of planets. The author develops the basic properties of almost periodic functions and presents the Bohr approximation theorem. This theorem can be considered as a justification of the method of epicycles used by Hipparchus and Ptolemy. Then he presents the results of ancient astronomers on the motion of the moon, the sun and the planets. The proof of the Bohr theorem is given in Appendix to Chapter 1, using the Peter—Weyl theorem, for which a proof based on the existence of an invariant mean on almost periodic functions is elaborated. The existence of such a mean is proved by a method of A. Weil reducing the problem to construct the Haar integral on compact groups. The existence of the Haar integral on compact groups is proved by using a version of the mean ergodic theorem. For this theorem and for the individual ergodic theorem a proof is also given. E. Hopf's maximal ergodic theorem is proved by the method of Garsia. In the Appendix the existence of the mean motion of a planet, represented by a non-harmonic trigonometric polynomial, is also proved by the Kronecker—Weyl theorem. The Appendix contains also Wintner's formula.

As the author writes in the Introduction: "the main purpose of Chapter II is to show how number theoretical considerations (known as 'small divisors') entered into the field of Celestial Mechanics". This chapter begins with Kepler's investigations and Kepler's elliptic motion. — According to the opinion of the author, Kepler's work was the turning point in the development of physics and astronomy. — A sketch of the Lagrange's method for the three body problem and that of Hill's lunar theory is given. Hill's theory is, in a sense, the origin of the qualitative theory of differential equations, a good bit of functional analysis, and a substantial portion of algebraic topology. The book ends with a sketch of the general theory of relativity and its most famous consequence in astronomy: the advance of the perihelion of Mercury.

It is remarkable what the author writes about the practical importance of the classical results of astronomy. Since the advent of radar astronomy and space probes we can give, with the aid of high speed computers, far more accurate predictions of planetary positions than anything provided by classical perturbation theory. However, the qualitative predictions of perturbation theory are still of importance in plasma physics. (These results will be given in Part II.)

The book is very well readable, and in spite of its relatively small size it gives a valuable insight into the history of the astronomy (in this Part up to the beginning of the 20th century).

The work is developed from lecture notes for a course given at Harvard University during the spring of 1968. Prerequisite is only the standard material in an introductory analysis course, except the section on general relativity where some familiarity with differential geometry is also required. The book can be used as a guide to a graduate-level course in Celestial Mechanics.

J. Szűcs (Szeged)

Jean-Luis Krivine, Théorie axiomatique des ensembles (Collection „Les précis de l'enseignement supérieur: Le mathématicien"), 120 pages, Paris, Presses Universitaires de France, 1969.

This is a small pocket-size book, requiring only little preliminary knowledge from logic, that provides an excellent introduction into axiomatic set theory. Its main concern is to present several classical consistency and independence proofs that open up the way towards the study of modern results due to Cohen and his followers. The reader may find the relative consistency proofs of the axiom of foundation, of the assertion that each cardinal is accessible, of the axiom of choice, and finally of the continuum hypothesis. Also, the independence of the axiom of infinity and that of the axiom of choice are verified; the latter only in case when the axiom of foundation is omitted, and the proof utilises a model of Fraenkel and Mostowski.

A. Máté (Szeged)

Joel W. Robbin, Mathematical logic. A first course (University Mathematics Series), xii+212 pages, New York—Amsterdam, W. A. Benjamin, Inc., 1969.

Apart from serving as an introductory textbook on mathematical logic for individual readers who are supposed to have high mathematical maturity but no specified preliminary knowledge, the book intends to provide a guideline for courses in logic for various ranges of people who might have some interest in the subject; these are students of mathematics, philosophy, linguistics, computer science and engineering. Since the reasons why these people are interested in logics are different, istructions are given how the book can be used for such a course, what selections of the topics might be preferred for various purposes. From the mathematician's point of view the clear style that renders possible a wider use of the work is in no way a disadvantage. In order to engage the interest of the readers with higher mathematical aspirations, the excercises, some of them quite difficult, provide the material for a more profound study of the subject. To the more difficult excercises the answers can be found at the end of the book. To make the task of the readers not trained in mathematics easier, the work contains an Appendix so as to make it self-contained. At the end, under the title "suggested reading", an excellent guide is given to the more interested students who want to specialize or deepen their knowledge in various parts of logics.

The six chapter headings are: 1. Propositional calculus, 2. First-order logic, 3. First-order recursive arithmetics, 4. Arithmetization of syntax, 5. The incompleteness theorems and other applications of the liars paradox, 6. Second-order logic.

Though the unification of such diverse purposes as the author attempted was not a simple task, we feel that, to say the least, the mathematics student reading this book will not suffer in any way from the fact that the author does not appeal exclusively to him.

A. Máté (Szeged)

J. T. Marti, Introduction to the theory of bases (Springer Tracts in Natural Philosophy, Vol. 18), IX+149 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1969.

In his famous book "Théorie des opérations linéaires", BANACH raised the question whether or not every separable Banach space possesses a basis. Although this problem has not yet been solved, there are at present a few hundred publications in existence on the theory of bases. This tract is an excellent attempt to collect and systematize the essential results of the subject.

The book consists of nine chapters, a comprehensive and exhaustive bibliography, and an author and a subject index. The reader is supposed to be familiar with the notion of linear vector spaces, algebras, etc., and with some fundamental theorems such as the Hahn—Banach extension theorem, the so-called "resonance theorem" of BANACH and STEINHAUS, the theorem of ALAOGLU on the compactness in the weak topology of the unit ball of a conjugate space, etc.

By the nature of the problem, the methods of proof used in the theory of bases are those of functional analysis. Chapter 1 is devoted to listing these basic definitions and facts. The theorems are given here without proofs, and the reader is referred to one of many good introductions to functional analysis.

For a vector space X of infinite dimension the concept of basis is closely related to the convergence of the series that uniquely correspond to each point of X. Accordingly, Chapter 2 deals with the relations among different types of convergence of series in Banach spaces. The definitions and properties of the most important types of bases for Banach spaces can be found in Chapter 3, together with examples of bases in several well-known spaces. Two sections are devoted to shrinking, boundedly complete and unconditional bases, and interrelations among them. Chapter 4 concernes the connections of bases, projections and orthogonality, as well as equivalent bases for Banach spaces.

Chapter 5 discusses the conclusions that can be drawn about the structure of X if one assumes the existence of a basis of a certain type for X. The assertions concern weak sequential completeness, separability, reflexivity, dimension (finite or infinite), etc. Chapter 6 takes up bases for Hilbert spaces.

Chapter 7 contains a generalization of the concept of basis for a Banach space, or more generally, for an F-space X, in which one considers a sequence of linear subspaces of X instead of a sequence of elements of X. In this way one obtains decompositions of these spaces which exhibit some properties similar to those of the bases.

Chapter 8 presents fine applications of both bases and decompositions to the theory of Banach algebras. These applications include, among others, the approximation problem of compact linear operators by linear operators of finite rank, and the theory of proper π-rings.

In the final chapter further generalizations of bases are treated for arbitrary topological vector spaces via discarding the idea of a series expansion and considering instead certain biorthogonal systems $\{x_\lambda, f_\lambda\}$ as generalized bases or dual generalized bases, respectively. It is worth noting that the basis problem is now answered for separable locally convex linear topological spaces in the negative direction.

The presentation of the book is concise but always clear and well-readable. It will be useful both for advanced mathematicians as a monograph and for able students as an excellent introduction to the theory of bases.

Ferenc Móricz (Szeged)

Underwood Dudley, Elementary number theory, IX+262 pages, San Francisco, W. H. Freeman and Co., 1969.

This work is an introduction to some elementary chapters of number theory. No special preliminary mathematical knowledge is needed for its reading (except for sections 21 and 22), therefore it is very useful for students intending to get acquainted with classical results of number theory. The mentioned sections contain the Mills formula which provides all prime numbers as well as a weakened form of the prime number theorem.

The book is well readable, due to the simple and easy-flowing style of the author. Let us mention that he gives short information on the history of the treated problems. In addition, the book contains over a thousand exercises (the more difficult problems are given with hints and solutions). These properties make possible for teachers of mathematics to use this book effectively in their work.

B. Csákány (Szeged)

Mathematische Hilfsmittel des Ingenieurs. Herausgegeben von **R. Sauer** und **I. Szabó**, unter Mitwirkung von H. Neuber, W. Nürnberg, K. Pöschl, E. Truckenbrodt und W. Zander, Berlin—Heidelberg—New York, Springer-Verlag.

Teil I (1967), verfaßt von G. Doetsch, F. W. Schäfke und H. Tietz, XV+496 Seiten, 103 Abbildungen. *Inhalt:* Komplexe Zahlen, Theorie der komplexen Funktionen, konforme Abbildung; spezielle Funktionen (Gamma-Funktion; Zylinder- und Kugelfunktionen, orthogonale Polynome, Mathieusche Funktionen, usw.); Fourier-, Laplace- und Mellin-Transformationen, einseitige und endliche Form dieser Transformationen (mit Tabellen); Theorie der Distributionen.

Teil II (1969), verfaßt von L. Collatz, R. Nicolovius und W. Törnig, XX+684 Seiten, 148 Abbildungen. *Inhalt:* Gewöhnliche Differentialgleichungen (auch im Komplexen) mit numerischen Lösungsmethoden, partielle Differentialgleichungen erster und zweiter Ordnung; Randwertaufgaben bei gewöhnlichen Differential- und Integralgleichungen, sowie bei einigen Typen partieller Differentialgleichungen; Potentialprobleme; Eigenwertaufgaben bei Differential- und Integralgleichungen, exakte und Näherungsverfahren zur Lösung solcher Aufgaben; Grundbegriffe und Anwendungen der Variationsrechnung.

Teil III (1968), verfaßt von T. P. Angelitch, G. Aumann, F. L. Bauer, R. Buliersch, H. P. Künzi, H. Rutishauser, K. Samelson, R. Sauer, I. Stoer, XVIII+534 Seiten, 101 Abbildungen.

Inhalt: Algebraische Strukturen und Gleichungen (hier sind die Sätze 1.10.13. und 3.1.8. unrichtig), lineare Algebra; affine und projektive Geometrie, Nomographie; sphärische Trigonometrie; Vektoralgebra und -analysis, Differentialgeometrie der Kurven und Flächen, Tensorkalkül; Interpolation durch Polynome und durch rationale Funktionen; numerische Quadratur; Approximation von Funktionen; Darstellung von Funktionen durch Rechenautomaten; lineare Optimierung (mit Ausblick in die nichtlineare); Rechenanlagen.

Teil IV in Vorbereitung.

Der rasche Fortschritt der Technik hat dazu geführt, daß für die Bearbeitung technischer Probleme immer umfassendere mathematische Hilfsmittel benötigt werden. Das vorliegende, auf vier Bände angelegte Werk behandelt diejenigen mathematischen Disziplinen, die für die Ingenieurpraxis von Bedeutung sind oder von Bedeutung zu werden versprechen. Obwohl es in erster Linie auf die Bedürfnisse der Ingenieure ausgerichtet ist, können es auch Mathematiker und Naturwissenschaftler mit Nutzen verwenden.

In jeder der behandelten Disziplinen bringt das Werk nicht nur die erforderlichen Formeln, sondern auch die grundlegenden Definitionen und Sätze und zwar in einer anschaulichen Darstellung; gute Abbildungen erleichtern das Verständnis der mehr abstrakten Begriffe und Sätze. Es wird stets erläutert, aus welchen physikalischen oder technischen Gründen das betreffende mathematische Problem und die eingeführten Begriffe entstammen. Beweise werden nur dann gebracht, wenn sie zum tieferen Verständnis eines Satzes oder eine Methode notwendig sind. Um die Anwendung der mathematischen Theorien auf die Ingenieurpraxis zu zeigen, werden auch einige konkrete technische Probleme ausführlich behandelt.

In jedem Kapitel sind die konkreten numerischen Methoden in den Vordergrund gestellt. Die Genauigkeit der Näherungsmethoden wird durch Tabellen veranschaulicht, in denen die genauen und die durch Annäherung gewonnenen Werte miteinander verglichen werden. In der Ingenieurpraxis sind zahlreiche Rechenverfahren allgemein gebräuchlich, die nur unter gewissen Bedingungen richtig sind; solche Verfahren werden hier mit mathematischer Genauigkeit diskutiert und die Bedingungen, bei denen das betreffende Verfahren ein richtiges Resultat ergibt, werden festgestellt. Stehen mehrere Methoden für die Lösung desselben Problems zur Verfügung, so werden die verschiedenen Methoden meistens aus dem Standpunkte der Vorteile und Nachteile beim tatsächlichen Ausrechnen untersucht. Auch die ALGOL-Programme der wichtigsten Methoden sind angegeben.

Einige grundlegende Begriffe werden in mehreren Kapiteln, von verschiedenen Standpunkten aus untersucht. In solchen Fällen streben die Verfasser auf keine gekünstelte Uniformisierung der Terminologie, sondern weisen sie auch auf die anderartigen Bezeichnungen oder Benennungen hin.

Am Ende jedes Kapitels findet sich ein reichliches Literaturverzeichnis, das die weitere Orientierung ermöglicht.

G. Szász (Budapest)

K. Zeller—W. Beekmann, Theorie der Limitierungsverfahren (Ergebnisse der Mathematik und ihre Grenzgebiete, Band 15), XII+314 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

Dieses Buch ist die zweite, erweiterte und verbesserte Auflage der Monographie von K. Zeller, die im Jahre 1958 mit demselben Titel veröffentlicht wurde. Die erste Auflage wurde in diesen *Acta*, **22** (1961), 152—153, ausführlich referiert.

Im originellen Text wurden — ohne wesentliche Änderungen — gewisse Verbesserungen vorgenommen. Die Bibliographie wurde mit zahlreichen neuen Literaturangaben aus den Jahren

1956—1968 ergänzt. (Dieses ergänzende Schriftenverzeichnis füllt 43 Seiten aus!) Weiterhin, am Ende des originellen Textes ist ein von W. BEEKMANN stammendes neues Kapitel (27 Seiten) hinzugefügt, in welchem die neueren Entwicklungen skizziert sind.

Die Ergänzungen geben dem Leser einen raschen Überblick über die in der Periode 1956—1968 in den verschiedenen Teilgebieten der Theorie der Limitierungsverfahren erzielten Fortschritte. Die originelle Zielsetzung und Betrachtungsweise des Buches sind bei der neuen Auflage unverändert geblieben.

Károly Tandori (Szeged)

M. Gross and A. Lentin, Introduction to Formal Grammars, IX+231 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1970.

The theory of normal grammars and formal languages was initiated one and a half decade ago by NOAM CHOMSKY and it has since then developed rapidly. Formal grammars are in fact special kinds of semi-Thue systems, playing an important rôle in the study of the structure of natural languages as well as in the construction of various kinds of programming languages.

The work of M. GROSS and A. LENTIN is an english translation of the book *Notions sur les grammaires formelles* (Gauthier—Villars, Paris, 1967), and it can serve both as an excellent introductory course and as a survey on formal grammars and formal languages. It consists of three parts and an Appendix.

The first part contains the algebraic and logical preliminaries. Chapter I defines a formal language as a part of a free monoid and a grammar as an algorithm which allows the words of a language to be enumerated. Chapter II introduces the concept of formal system and illustrates it by a presentation of a formalized variant of the propositional calculus. Chapter III discusses different types of productions and deals in general with semi-Thue, Thue and normal systems. Chapter IV illustrates the rôle of algorithms in mathematical linguistics and introduces the concepts of computability and Turing machine. Chapter V is devoted to the operations on computable functions, Gödel's techniques, recursive stes, recursively denumerable sets and the equivalence of computability and recursiveness. Chapter VI shows that every recursively denumerable set can be generated by a combinatorial system.

The second part is devoted to the study of some important classes of formal languages. The row is opened by Chapter VII, where the class of context-free grammars and languages is defined and some decidable properties of them are presented: such are e.g. the property of the generated language being empty, finite or infinite, and the property of a sentence belonging to the given language or not. Here are also discussed some closure properties of context-free languages with respect to several simple operations studied. Chapter VIII deals with a few undecidable properties of context-free grammars. Chapter IX establishes that the class of context-free languages is identical to the class of languages accepted by push-down automata. Chapter X formulates the well-known connection between regular languages and finite automata, and Chapter XI is devoted to the study of languages obtained by the solution of different kinds of interesting systems of equations. Finally, Chapter XII deals with the context-sensitive grammars and proves the Kuroda—Landweber theorem: the class of languages generated by context-sensitive grammars coincides with the class of languages accepted by non-deterministic linear bounded automata.

The third part contains further, purely algebraic, studies about formal grammars and languages. Chapter XIII sets forth some concepts of semigroup theory. Chapter XIV gives algebraic characterizations of regular languages, and Chapter XV presents the fundamental theorem for context-free languages, according to which every such language can be obtained by a homomorphism from a suitably chosen standard context-free language. Chapter XVI contains applications of formal power series to the study of formal languages.

Appendix deals with transformational grammars and some problems related to transformations.

As was remarked above, the book has an overwhelmingly introductory character. It reports many facts without proofs and refers the reader to the appropriate source in the bibliography.

Students of higher courses, programmers and researchers in mathematical linguistics will find the work very valuable. It is of great interest also for those in other fields of mathematics wanting to consult basic results on formal languages.

I. Peák (Szeged)

H. Cramér, Random variables and probability distributions (Cambridge Tracts in Mathematics and Mathematical Physics, No. 36), third edition, IX+118 pages, Cambridge University Press, 1970.

This tract, now in its third edition, is devoted almost entirely to problems connected with the Central Limit Theorem and some of its generalizations and modifications in various directions. The subject is developed in a pure-mathematical aspect of probability theory, without any reference to applications. It should perhaps be recalled that the book, when originally published, was one of the earliest works that were concerned with questions of this field in the light of the axiomatic foundations introduced by A. KOLMOGOROFF in his book "Grundbegriffe der Wahrscheinlichkeitsrechnung".

The book consists of three parts amounting to ten chapters, a list of abbreviations and notations, a short but effective bibliography supplemented with the data of some recent works.

The first part contains the axioms and preliminary theorems of probability theory, treating it as a branch of the theory of completely additive set functions.

The second part deals in great detail with distributions of sums of the type $Z_n = X_1 + \ldots + X_n$, where the X_k are independent random variables, including the most important theorems of J. W. LINDEBERG, P. LÉVY, W. FELLER, the author, etc. The contents of Chapter VII are fundamental for the theory of error estimation and asymptotic expansions. This part of the first two editions of the book contained Liapounoff's classical inequality. For the third edition this chapter has been partly rewritten, and now brings a proof of the sharper inequality due to BERRY and ESSEN. In Chapter VIII some theorems of the preceding chapters are extended to stochastic processes with stationary and independent increments.

The object of the third part is to generalize the results obtained for distributions in R_1 to R_k. The author restricts himself to a brief discussion of some typical generalizations of this kind.

The book is well readable, and there are several valuable historical comments in the footnotes. It is warmly recommended to everybody who intends to become familiar, with a comparatively sparing effort, with the most essential notions and results of the field known as the Central Limit Theorem.

Ferenc Móricz (Szeged)